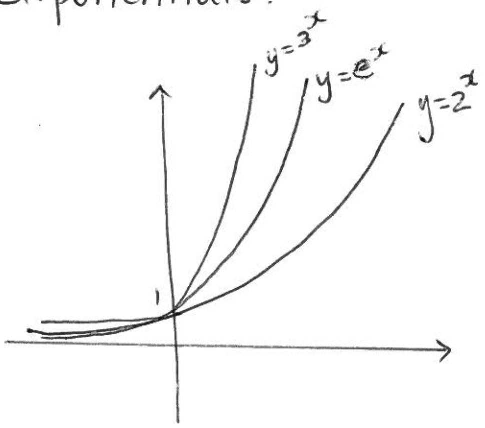
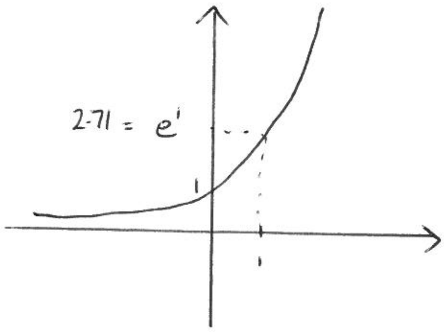
More on Logs + Exponentials

Refresher : Exponentials.

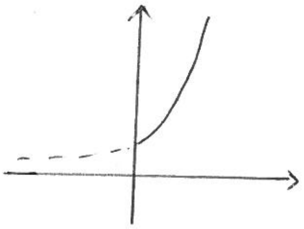


- Grows (or decays) rapidly
- Remember properties
  - y int = 1
  - horiz asymptote x axis
  - always positive

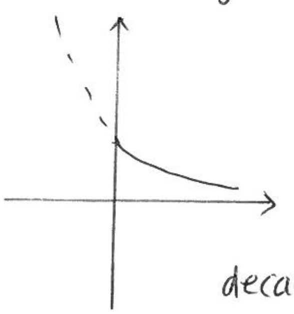


$y=e^x$ is a function
(remember input-output)

- Growth + decay can be modelled by exponentials



growth



decay

$y = Ae^{kt}$

- we need to know how to work with exponentials
  - remember power rules.

eg 1a) Simplify $64^{-1/6} = \frac{1}{64^{1/6}} = \frac{1}{2}$

b) Simplify $2^{3x} \times (\sqrt{8})^{x}$

$= 2^{3x} \times (\sqrt{2^3})^{x}$

$= 2^{3x} \times (2^{3/2})^{x}$

$= 2^{3x + \frac{3x}{2}}$

$= 2^{9x/2}$

c) Solve $\left(\frac{1}{3}\right)^{x} \times 27 = 9^{x}$

$3^{-x} \times 3^{3} = 3^{2x}$

$3^{-x+3} = 3^{2x}$

$\therefore -x+3 = 2x$

$3 = 3x$

$\therefore x = 1$

d) Simplify $(32x^5)^{2/5} \times \left(\frac{25}{x^4}\right)^{1/2}$

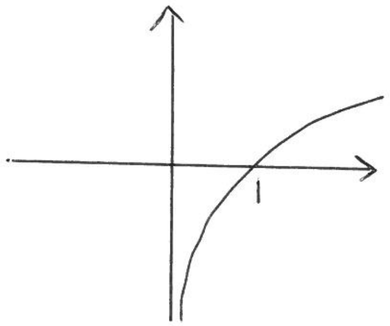
$= 32^{2/5}\, x^{2} \times \frac{25^{1/2}}{(x^4)^{1/2}}$

$= (32^{1/5})^{2}\, \cancel{x^{2}} \times \frac{5}{\cancel{x^{2}}}$

$= 2^{2} \times 5$

$= 20$

Logs : $y = \log_e x$



. Logs undo exponentials.

. Remember properties.

- $\log 1 = 0$
- can't take log of neg + 0.

- Remember log rules + how to work with logs.

eg2a) Find the exact value of $\log_5\left(\frac{1}{\sqrt{125}}\right)$

$$= \log_5\left(\frac{1}{\sqrt{5^3}}\right)$$

$$= \log_5\left(\frac{1}{5^{3/2}}\right)$$

$$= \log_5 5^{-3/2}$$

$$= -3/2$$

eg b) Simplify $\left(\log\left(\frac{u^3}{v}\right) + \log\left(\frac{v}{u}\right)\right) \div \log\sqrt{u}$

$$= \log\left(\frac{u^3}{v} \times \frac{v}{u}\right) \div \log\sqrt{u}$$

$$= \log u^2 \div \log u^{1/2}$$

$$= 2\log u \div \frac{1}{2}\log u$$

$$= \frac{2\log u}{1/2 \log u}$$

$$= 4$$

c) Solve $2\ln x = \ln(x+6)$

ie: $\ln x^2 = \ln(x+6)$

ie: $x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

Ignore neg (since log can't take negs)

$\therefore$ Soln is $x = 3$.

d) Solve $7^x = 12$

$\log 7^x = \log 12$

$x \log 7 = \log 12$

$x = \dfrac{\log 12}{\log 7}$

Since we know about the relationship between logs + exp we can now find powers.

e) Solve $e^{9x-1} = 11$

$\log e^{9x-1} = \log 11$

ie: $9x - 1 = \log 11$

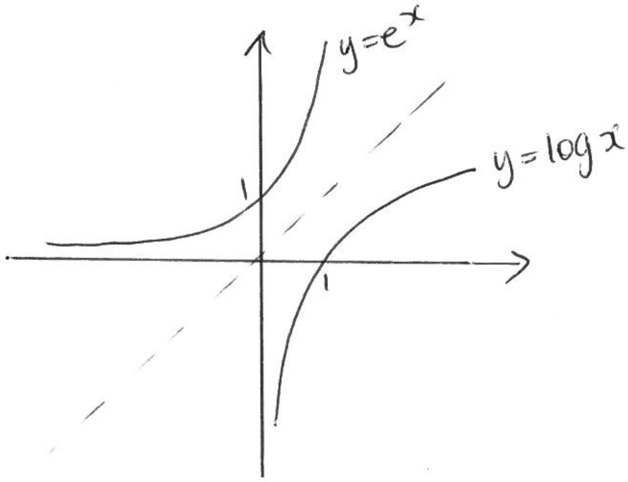
$9x = \log 11 + 1$

$x = \dfrac{\log 11 + 1}{9}$

Special Relationship of Exp + Logs

- They undo each other
- ie: They are inverses.
- The proper definition of a log is that it is the inverse of the exponential.

- Geometrically inverses have a special propety



We can say $f(x) = e^x$ has Dom $= \mathbb{R}$
Range $= (0,\infty)$

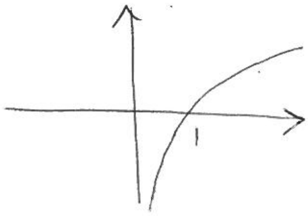
and $f^{-1}(x) = \log x$ has Dom $= (0,\infty)$
Range $= \mathbb{R}$

- We now know what the graph of $y = \log x$ looks like.

Modifying the Log function

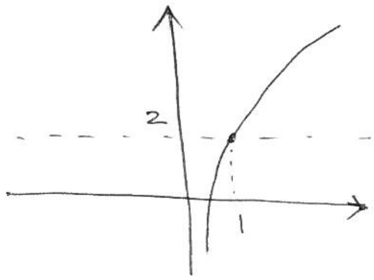
eg3)

$y = \log x$



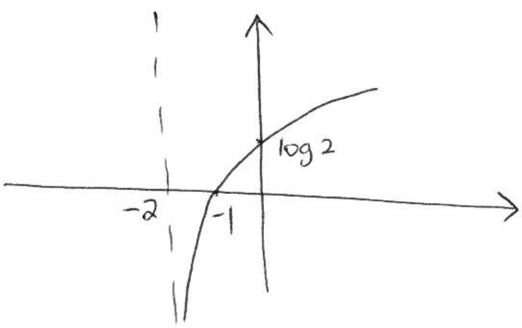
- grows slowly
- cuts x axis at 1
- vertical asymptote y-axis

a) $y = \log x + 2$



vertical shift up to 2

b) $y = \log(x+2)$

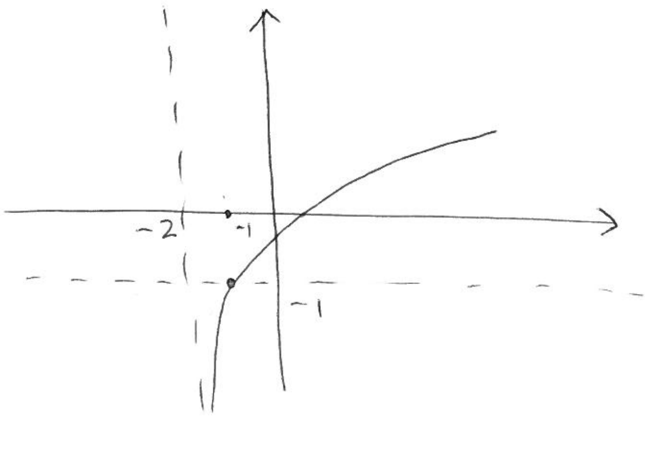


horizontal shift
what happened at 0 now happens at -2

y int $\rightarrow$ Let $x = 0$

$y = \log 2$

c) $y = \log(x+2) - 1$



x int $\rightarrow$ Let $y = 0$

$\log(x+2) - 1 = 0$

$\log(x+2) = 1$

$x + 2 = e^1$

$x = e^1 - 2$

≈ 0.7

Dom $= (-2, \infty)$

Range $= \mathbb{R}$